

NUTRIENT TRANSPORT INSIDE AND OUTSIDE THE ROOT RHIZOSPHERE: GENERALIZED MODEL¹

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ABSTRACT

An earlier article briefly reviewed the possible effects of the rhizosphere on nutrient transport to a root. In that article, transport in the rhizosphere was considered separately from bulk transport with coupling arising from the rhizosphere bulk-soil boundary conditions. Four linear nutrient transport problems were considered and solved analytically.

In this article we have extended the results of the earlier article to account for nonlinear root boundary conditions; arbitrary spatial and temporal changes in the buffer power and diffusion coefficients; nonlinear sources and sinks of nutrient in both the rhizosphere and bulk soil; and arbitrary, nonlinear, initial conditions.

The computer results indicate that if the buffer power, effective diffusion coefficients, and sources or sinks differ between the rhizosphere and bulk soil, an alteration of the concentration profile and total nutrient uptake may take place. Certainly, this effect can play a role in plant shoot development, as such development is directly affected by nutrient uptake.

The numerical model was calibrated with the analytical solutions to the linear problems presented in Cushman (1982). Although the analytical solutions agree with the numerical solutions, it is much less time consuming and expensive to use the numerical simulation. Moreover, the numerical solution is much more general than the analytical solution. If one determines the diffusion coefficient, buffer power, and source or sink as a function of moisture content, the numerical model can handle transient-state moisture conditions provided the moisture content does not change too rapidly. Such data at this point, however, are not available.

INTRODUCTION

In the first part of this sequence of articles (Cushman 1982), we presented a brief review of properties of the root rhizosphere and showed how these properties may affect the transport equations that describe movement of nutrient to a single root. The analysis of the transport process was characterized by the subdivision of the root-soil environment into three concentric regions (1) the root, (2) a cylindrical rhizosphere, and (3) the bulk-soil matrix exterior to the rhizosphere. Transport in the rhizosphere and the exterior bulk-soil was represented by two linear nondimensional problems, each of

which was solved analytically. The solutions were composed of infinite series of negative fractional order Bessel functions of the first and second kind in space and exponential functions in time. No graphical results were presented to illustrate the mathematical results.

As is well known (see Hodges 1974 or Barber and Silverbush 1983), except for low nutrient (or very high nutrient) concentration, root absorption of nutrient is nonlinear. Generally, it is assumed to follow a Michaelis-Menton type of relationship. Because of this nonlinear relationship, the analytical solutions of Cushman (1982) are limited to the regions mentioned. To account for the intermediate range of concentrations, we will include the general Michaelis-Menton root surface condition in this article.

As mentioned, in the Cushman (1982) article,

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the flow domain was subdivided into three regions. This subdivision may be a reasonable approximation in some cases, but the obvious generalization, to account for many more flow situations, would be to allow for arbitrarily varying (with space and time) buffer power and diffusion coefficients. The main reason for allowing variations in buffer power in the vicinity of the root is the ability of the root to alter pH in its immediate environment. There are several possible definitions for the buffer power; we suggest the reader consult the appendix for a discussion. Because the root gives off mucigels and because the root alters the soil bulk density in its vicinity, the diffusion coefficient will vary with radial distance from the root as well. This variation of the diffusion coefficient can be modeled as a discontinuous or continuous phenomenon. In this article we allow for a general variation in the buffer power and diffusion coefficient.

Another limitation of the Cushman (1982) article, as well as earlier articles, is the reliance on exponential root growth. As is generally known, the assumption of exponential growth is valid only for relatively young roots. In this article we allow for polynomial, as well as exponential, root growth rates.

The general transport problem may contain nonlinear sources or sinks of nutrient (e.g., due to microbial activity). To account for these nonlinear sources or sinks, we introduce (purely as an example) polynomial sources and sinks. In the model we will develop, however, any nonlinear source or sink will work.

Finally, in this article we present a numerical algorithm to solve the fully nonlinear problem that is second order in space-time, unconditionally stable, and quadratically convergent. Though the algorithm, including Newton's methods, presented is similar to those used in the fluids literature, it does not appear in the soils literature. The scheme can be rigorously shown to converge to the true solution of the partial differential equation (PDE) in the second order.

In this article we present only numerical results. Experimental results that include the rhizosphere are not available, due to the difficulty in constructing accurate experimental techniques, to study the rhizosphere and its effect on the effective diffusion coefficient and buffer

power. The graphical results and programs should, however, be of value in interpreting and predicting the effects the root rhizosphere has on nutrient uptake.

GENERALIZED MODEL

Although the transport equations of Cushman (1982) would probably model the rhizosphere bulk-soil system reasonably well, a more accurate model would allow for a rhizosphere that gradually changes into bulk soil with distance from the root, i.e., there would not be a double ring model, but rather a model with smoothly varying radial functions. Moreover, both the buffer power and effective diffusion coefficients may vary with time in the rhizosphere (e.g., due to variations in moisture content). Throughout this article we let r_0 be the root radius, C_l the concentration in solution, b , the buffer power, D the effective diffusion coefficient, v_0 the pore water velocity at the root, α the source or sink, k the root absorption power, J_{\max} the maximum influx, E the efflux, and r_1 the outer radius of influence of the root. To include these generalizations in our model, we examine the distributional equation

$$b(r, t) \frac{\partial C_l}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[r b(r, t) D(r, t) \frac{\partial C_l}{\partial r} + v_0 r_0 C_l \right] + \alpha(r, t, C_l) \quad (1)$$

subject to

$$D(r_0, t) b(r_0, t) \frac{\partial C_l}{\partial r} + v_0 C_l = 0 \quad (2)$$

$$= \frac{k C_l}{1 + \frac{k}{J_{\max}} C_l} - E, \quad r = r_0, t > 0$$

$$D(r_1, t) b(r_1, t) \frac{\partial C_l}{\partial r} + \frac{r_0}{r_1} v_0 C_l = 0, \quad r = r_1, t > 0, \quad (3)$$

and

$$C_l(r) = f(r), \quad t = 0, \quad r_0 \leq r \leq r_1 \quad (4)$$

where Eq. (2) will be recognized as the Michaelis-Menton condition.

If we introduce the nondimensional variables

$$a = r_1/r_0$$

$$\eta = tD^{(1)}/r_0^2$$

$$\mu = r/r_0$$

$$\theta = 1 - C_l/C^{(1)}$$

$$D'(\mu, \eta) = \frac{D}{D^{(1)}} \Big|_{(\mu, \eta)}$$

$$\nu(\mu, \eta) = -r_0 v_0 / 2D^{(1)}b$$

$$Q'(\mu, \eta, \theta) = \frac{-\alpha r_0^2}{C^{(1)}D^{(1)}b} \Big|_{(\mu, \eta, \theta)}$$

$$\tau = k/v_0$$

$$d' = E/kC^{(1)}$$

$$\rho' = kC^{(1)}/J_{\max}$$

where $D^{(1)} = D(r_0, 0)$ and $C^{(1)} = C_l(r_0, 0)$, then we find

$$b \frac{\partial \theta}{\partial \eta} = \frac{\partial}{\partial \mu} \left(b D' \frac{\partial \theta}{\partial \mu} \right) + \frac{b}{\mu} (D' - 2\nu) \frac{\partial \theta}{\partial \mu} + b Q' \quad (5)$$

$$\frac{\partial \theta}{\partial \mu} = \frac{2\nu(1-\theta)}{D'} \left[\frac{\tau}{1 + \rho'(1-\theta)} - 1 \right] - \frac{2\nu d' \tau}{D'}, \quad \mu = 1, \quad \eta > 0 \quad (6)$$

$$\frac{\partial \theta}{\partial \mu} = \frac{-2\nu}{D'a} (1-\theta), \quad \mu = a, \quad \eta > 0 \quad (7)$$

and

$$\theta = 1 - \frac{C_l}{C^{(1)}}, \quad 1 \leq \mu \leq a, \quad \eta = 0 \quad (8)$$

For simplicity we will consider only the interroot competition problem (Eq. (7)).

To verify that Eqs. (5) through (8) reduce to the equations of Cushman (1982) (under the assumptions and notation of that article), we consider Eq. (5) in the sense of distributions (Donoghue 1969). Setting

$$D = D^{(1)}\chi_{(1,a^{(1)})} + D^{(2)}\chi_{(a^{(1)},a^{(2)})}$$

$$b = b^{(1)}\chi_{(1,a^{(1)})} + b^{(2)}\chi_{(a^{(1)},a^{(2)})}$$

$$Q' = Q\chi_{(1,a^{(1)})} \quad (9)$$

$$\begin{aligned} \nu &= -\frac{r_0 v_0}{2D^{(1)}[b^{(1)}\chi_{(1,a^{(1)})} + b^{(2)}\chi_{(a^{(1)},a^{(2)})}]} \\ &= \nu^{(1)}\chi_{(1,a^{(1)})} + \frac{\nu^{(2)}}{\lambda}\chi_{(a^{(1)},a^{(2)})} \end{aligned}$$

we find that both Eqs. (14) and (19) of Cushman (1982) result. Moreover, noting that the distributional derivative of a characteristic function (χ_A) of a set A may be written as a sum of Dirac delta functions, we also get Eq. (17) (Cushman 1982). The other boundary and initial conditions are obvious, e.g., setting $\rho' = 0$ in Eq. (6) produces Eq. (15) of Cushman (1982) for linear uptake. Henceforth, for our model problem of nutrient transport, we will use Eqs. (5) through (8) with b and D' arbitrary functions of μ and η ; also we may take Q' nonlinear in θ as well as an explicit function of μ and η .

Using Eq. (6) for our root boundary condition, we rewrite Eq. (29) of Cushman (1982) as

$$T = \frac{2\pi r_0^3 k C_i}{D^{(1)}} \left\{ L_0 \int_0^{\eta_{\max}} \left[\frac{1-\theta}{1+\rho(1-\theta)} - d \right] ds \right. \quad (10)$$

$$\left. + \int_0^{\eta_{\max}} \frac{dg}{d\eta} \int_0^{\eta_{\max}-\eta} \left[\frac{1-\theta}{1+\rho(1-\theta)} - d \right] ds d\eta \right\}$$

Here it should be noted that $dg/d\eta$ has units of length. If g is represented by an exponential

$$g(t) = L_0 \exp(\gamma t) \quad (11)$$

and the initial concentration is low ($\rho \approx 0$), then Eq. (10) reduces to Eq. (31) of Cushman (1982). If g is exponential and C_i is arbitrary, then Eq. (10) becomes

$$T = \frac{2\pi r_0^3 k C_i L_0}{D^{(1)}} T_N \quad (12)$$

$$\begin{aligned} T_N &= \int_0^{\eta_{\max}} \left[\frac{1-\theta}{1+\rho(1-\theta)} - d \right] ds \\ &+ \psi^{(1)} \int_0^{\eta_{\max}} e^{\psi^{(1)}\eta} \int_0^{\eta_{\max}-\eta} \left[\frac{(1-\theta)}{1+\rho(1-\theta)} - d \right] ds d\eta \end{aligned} \quad (13)$$

where

$$\psi^{(1)} = \frac{\gamma r_0^2}{D^{(1)}}$$

Another interesting case arises if the root has

polynomial growth rate. In this case we set

$$\begin{aligned} g(t) &= L_0 \left[1 + \sum_{i=1}^N \gamma_i t^i \right] \\ &= L_0 \left[1 + \sum_{i=1}^N \phi_i \eta^i \right] \end{aligned} \quad (14)$$

where

$$\phi_i = \frac{\gamma_i r_0^{2i}}{(D^{(1)})^i} \quad (15)$$

and

$$\frac{dg}{d\eta} = L_0 \sum_{i=1}^N i \phi_i \eta^{i-1} \quad (16)$$

Using Eq. (16) in Eq. (10), we get

$$\begin{aligned} T_N &= \int_0^{\eta_{\max}} \left[\frac{1 - \theta}{1 + \rho(1 - \theta)} - d \right] ds \\ &+ \sum_{i=1}^N i \phi_i \int_0^{\eta_{\max}} \eta^{i-1} \int_0^{\eta_{\max-\eta}} \left[\frac{1 - \theta}{1 + \rho(1 - \theta)} - d \right] ds d\eta \end{aligned} \quad (17)$$

where T is given by Eq. (12). We will use either Eq. (13) or (17) for the nondimensional total uptake equations.

DIFFERENCE EQUATIONS

There are numerous ways to difference Eq. (5); we use $0(\Delta\mu^2)$ spatial derivatives; a forward-in-time temporal derivative; and time average of θ_j . The resulting difference equation representing Eq. (5) is second order in space and time, unconditionally stable, and given by

$$\begin{aligned} 0 &= F_j(\bar{\theta}^{n+1}, \bar{\theta}^n) \\ &= \Delta\eta(A_j - \beta_j^{(1)})^{n+1/2} \theta_{j-1}^{n+1} \\ &+ [1 + \Delta\eta(\beta_j^{(1)} + \beta_j^{(2)})^{n+1/2}] \theta_j^{n+1} \\ &- \Delta\eta(A_j + \beta_j^{(2)})^{n+1/2} \theta_{j+1}^{n+1} \\ &+ \Delta\eta(A_j - \beta_j^{(1)})^{n+1/2} \theta_{j-1}^n \\ &- [1 - \Delta\eta(\beta_j^{(1)} + \beta_j^{(2)})^{n+1/2}] \theta_j^n \\ &- \Delta\eta(A_j + \beta_j^{(2)})^{n+1/2} \theta_{j+1}^n \\ &- \Delta\eta Q' \left(\mu_j, \frac{\theta_j^{n+1} + \theta_j^n}{2} \right) \end{aligned} \quad (18)$$

where

$$A_j = \frac{(D' - 2\nu)_j^{n+1/2}}{4\mu_j \Delta\mu}, \quad \beta_j^{(1)} = \frac{(bD')_j^{n+1/2}}{2b_j^{n+1/2} \Delta\mu^2},$$

and

$$\beta_j^{(2)} = \frac{(bD')_j^{n+1/2}}{2b_j^{n+1/2} \Delta\mu^2}$$

By addition of a node internal to the root we may derive second-order centered differences for the root boundary condition. The resulting difference scheme is

$$\begin{aligned} \theta_0^{n+1} &= G^{n+1/2} (1 - \theta_1^{n+1}) \\ &\cdot \left[\frac{\tau}{1 + \rho(1 - \theta_1^{n+1})} - 1 \right] \\ &+ H^{n+1/2} + \theta_2^{n+1} \end{aligned} \quad (19)$$

where

$$G^{n+1/2} = \frac{-4\Delta\mu\nu}{D'} \bigg|_1^{n+1/2}$$

and

$$H^{n+1/2} = -G^{n+1/2} d\tau$$

The outer boundary condition is obtained in a fashion identical to Eq. (19) as

$$\theta_{k+1}^{n+1} = \theta_{k-1}^{n+1} + P^{n+1/2} (\theta_k^{n+1} - 1) \quad (20)$$

where

$$P^{n+1/2} = \frac{4\gamma\Delta\mu}{D'a^{(2)}} \bigg|_k^{n+1/2}, \quad \mu = a$$

The reader should at this point note that Eq. (18) is nonlinear. We thus must use an iterative method to solve this system. The most straightforward method with quadratic convergence is Newton's method. The details of the implementation of Newton's method to our scheme are available from the author.

EXAMPLES AND COMPARISONS

To help in interpreting the data, we present all results in dimensional form. However, the reader should note that any other combination of dimensional variables that produce the equivalent nondimensional variables, will give the same results.

Throughout this article we will use soil parameters consistent with potassium in a silt loam.

We use plant parameters similar to those of corn (*Zea mays* L.): $r_0 = .015$ cm, $v_0 = 10^{-7}$ cm/s, $r_1 = .25$ cm, $J_{\max} = 7.05 \cdot 10^{-6}$ $\mu\text{moles}/\text{cm}^2$ s, $k = 1.28 \cdot 10^{-4}$ s $^{-1}$, and $E = 2.47 \cdot 10^{-8}$ $\mu\text{moles}/\text{cm}^2$ s. We set $C_i = .01$ μmoles unless otherwise noted, and total uptakes are given at 10 d.

We first examine the relationship between the linear-analytical solution of Cushman (1982) and the nonlinear-numerical solution. For this case only, we set $E = 0$; for all other graphs E is as defined above. The buffer power b is given by $b = 12$ for $r < .03$ cm and $b = 24$ for $r \geq .03$ cm. Here .03 cm is the radius of the rhizosphere (r'). We set $D = 3.47 \cdot 10^{-8}$ cm 2 /s and $C_i = .1$, .01, .001, and .0001 μmoles , respectively. Only the concentration profiles for $C_i = .1$, .01, and .001 are plotted in Fig. 1. The concentration profile for $C_i = .001$ of the numerical solution is identical to the analytical solution. It should be noted that at .74 d the concentration profiles are very similar for $C_i = .1$, .01, and .001 μmoles . An interesting comparison is that of total uptake. If we assume that the root is growing exponentially with $l = .0000016$ s $^{-1}$, the total uptake divided by $C_i L_0$ is 149.9, 155.7, 156.2, and 156.2 for $C_i = .1$, .01, .001, and .0001 μmoles , respectively. Thus when $C_i < .01$, there is very little difference in the nonlinear solution for total uptake and the linear solution. If we had assumed the roots were growing linearly at a rate L_0 ($1. + 0.03 t$), then the total uptake divided by $L_0 C_i$ would be $16.8 \cdot 10^3$, $17.4 \cdot 10^3$, $17.5 \cdot 10^3$,

and $17.5 \cdot 10^3$, respectively. Again it is seen that if $C_i < .01$ μmoles , the linear solution of Cushman (1982) is a good representation of the concentration profile and uptake.

We next examine variations in the radius (r') of a well-defined rhizosphere. We maintain b as above, but set the rhizosphere radius r' to .02, .03, and .045 cm, respectively. Graphical results of the concentration profile are presented in Fig. 2 at 0.74 and 10.0 days. As can be seen, the larger r' is, the larger the depletion of nutrient near the root is, but the smaller the depletion is outside the rhizosphere. If we had b greater in the rhizosphere than outside, just the opposite would have occurred. Suppose now that we let $b = 24$, be constant throughout the soil profile, but let $D = 6.94 \cdot 10^{-8}$ cm 2 /s for $r < r'$ and $D = 3.47 \cdot 10^{-8}$ cm 2 /s for $r \geq r'$ where r' is .02, .03, and .045, respectively. Results for this case are presented in Fig. 3. Now we see that as r' increases, the depletion in the rhizosphere is less, and the depletion of nutrient is greater outside the rhizosphere. Again, the opposite would hold if D were smaller in the rhizosphere than outside.

Suppose now that the rhizosphere is not well-defined, i.e., properties vary smoothly in the soil profile. As an elementary example, we will assume that $D = 3.47 \cdot 10^{-8}$ cm 2 /s and that $b = b_0(r - r')/2(r' - r_0) + b_0$ for $r < r'$ and that $b = b_0$ for $r \geq r'$ where $b_0 = 24$, and $r' = .02$, .03, and .045 cm, respectively. Figure 4 presents graphical results for this case. Note that because the

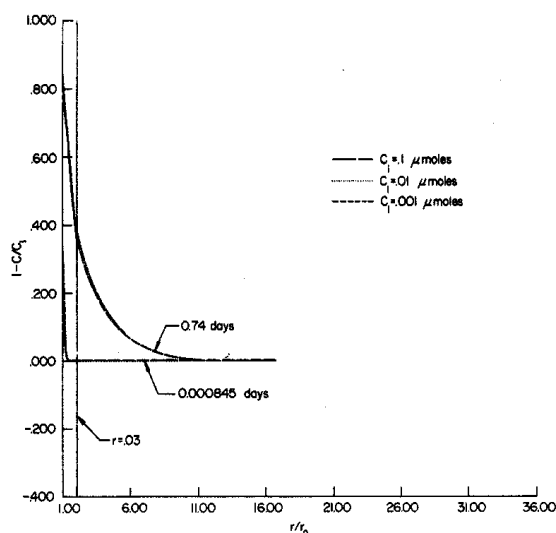


FIG. 1. A comparison of the analytical solution of Cushman (1982) to the numerical solution. The curve for $C_i = .001$ is identical to the linear solution.

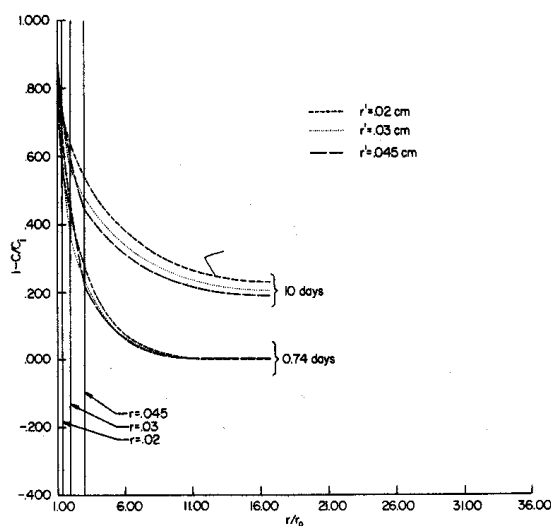


FIG. 2. A comparison of the effect of rhizosphere radius changes in a well-defined rhizosphere; $b = 12$ in the rhizosphere, and $b = 24$ outside the rhizosphere.

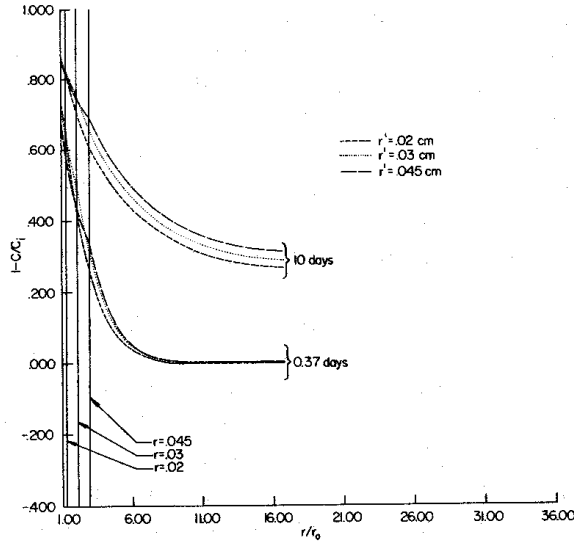


FIG. 3. A comparison of the effect of rhizosphere radius changes in a well-defined rhizosphere; $D = 6.94 \cdot 10^{-8} \text{ cm}^2/\text{s}$ in the rhizosphere and $D = 3.47 \cdot 10^{-8} \text{ cm}^2/\text{s}$ outside the rhizosphere.

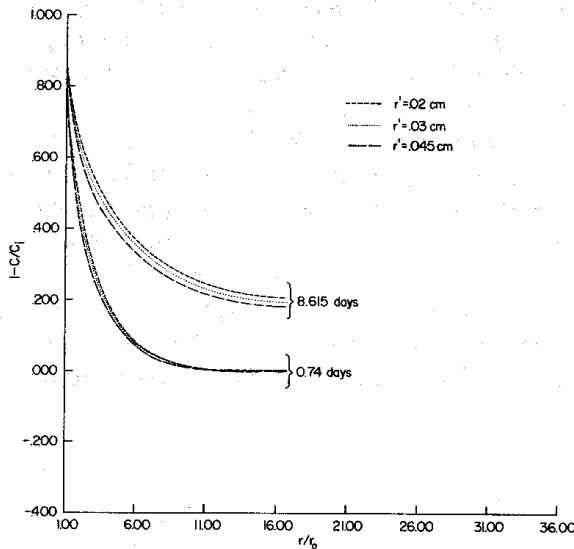


FIG. 4. A comparison of the effect of rhizosphere radius changes in a non-well-defined rhizosphere; $b = b_0(r - r')/2(r' - r_0) + b_0$ for $r < r'$, and $b = b_0$ for $r \geq r'$ where $b = 24$.

buffer power varies smoothly, there is no sharp break in the slope of the concentration profile. However, a comparison of Fig. 4 with Fig. 2 shows only a small difference between the two. Figure 5 is similar to Fig. 4, but now $b = 24$ throughout the profile, while $D = D_0(r - r')/(r_0 - r') + D_0$ for $r < r'$, and $D = D_0$ for $r \geq r'$ where $D_0 = 3.47 \cdot 10^{-8} \text{ cm}^2/\text{s}$. The characteristics of the concentration profile in Fig. 5 are similar to Fig. 3, but they are less dramatic in Fig. 5.

Let us now look at the effect a source or sink of nutrient has on nutrient transport in the soil system. We will confine our attention, for the sake of brevity, to a source term in the rhizosphere. For simplicity, we will assume the source term is a polynomial

$$\alpha = \sum_{i=1}^N \alpha_i C^{i-1}$$

or

$$Q' = \frac{-\alpha r_0^2}{C^{(1)} D^{(1)} b}$$

$$= \sum_{i=1}^N \xi_i (1 - \theta)^{i-1}$$

where

$$\xi_i = \frac{-r_0^2 \alpha_i (C^{(1)})^{i-2}}{D^{(1)} b}$$

are the nondimensional polynomial source coefficients. Again, for ease in interpreting the results, we use dimensional variables for the figures; however, other dimensional variables will produce the same results if the associated non-dimensional variables are equal. In Fig. 6, $b = 24$, and $D = 3.47 \cdot 10^{-8} \text{ cm}^2/\text{s}$ throughout the soil profile. The figure compares the profiles for the case of no source to the case where $\alpha_1 = 10^{-5}$, $\alpha_2 = 3.6 \cdot 10^{-5}$, $\alpha_3 = 1.27 \cdot 10^{-4}$, and $\alpha_4 = 4.55 \cdot 10^{-4}$ with $r' = .03 \text{ cm}$. It is interesting in this case to

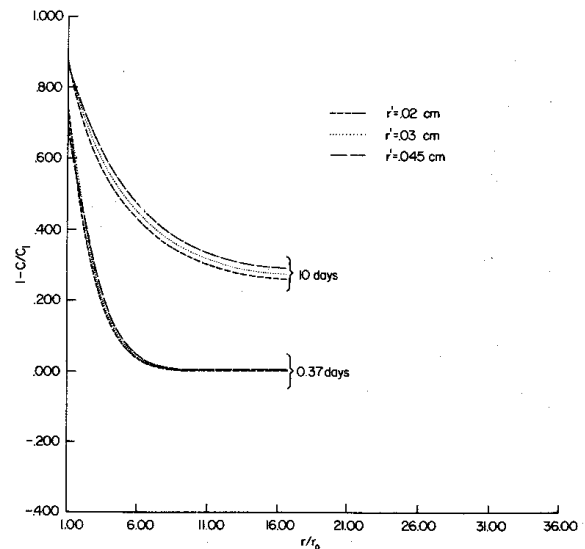


FIG. 5. A comparison of the effect of a variable rhizosphere radius in a non-well-defined rhizosphere; $D = D_0(r - r')/(r_0 - r') + D_0$ if $r < r'$, and $D = D_0$ if $r \geq r'$ where $D_0 = 3.47 \cdot 10^{-8} \text{ cm}^2/\text{s}$.

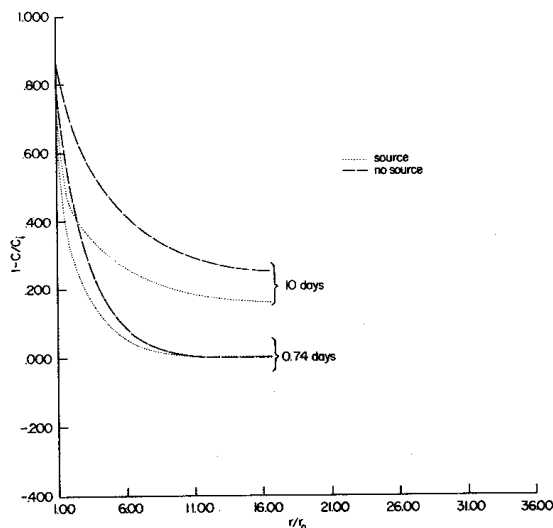


FIG. 6. The effect of a source in the rhizosphere on nutrient transport.

note that although there is a distinct rhizosphere, there is no break in the slope of the concentration profile as in previous figures.

SUMMARY

In this article we have extended the results of Cushman (1982) to account for nonlinear root boundary conditions, arbitrary spatial and temporal changes in the buffer power and diffusion coefficients, nonlinear sources and sinks of nutrient in both the rhizosphere and bulk soil, and arbitrary, nonlinear, initial conditions.

The graphical results in this article were the outcomes of five computer programs: RHIZ01, UPTAKE1, UPTAKE2, PLOT1, and PLOT2. RHIZ01 solves the general nonlinear transport problem for the concentration profile as a function of time and space. The program was written to handle data more general than the data used for the graphical results presented. UPTAKE1 and UPTAKE2 were written to compute total uptake for exponential and polynomial growth at specific times and as a function of time, respectively. PLOT1 and PLOT2 plot (to CRTs and hard copy) concentration versus distance at various times and total uptake versus time, respectively. All computer programs are available for distribution.

The computer results indicate that if the buffer power, effective diffusion coefficients, and sources or sinks differ between the rhizosphere and bulk soil, a profound alteration of the concentration profile and total nutrient up-

take may take place. Certainly, this effect can play a role in plant shoot development, as such development is directly affected by nutrient uptake.

The numerical model was calibrated with the analytical solutions to the linear problems presented in Cushman (1982). Although the analytical solutions agree with the numerical solutions, it is much less time consuming and expensive to use the numerical simulation. Moreover, the numerical solution is more general than the analytical solution. If one determines D , b , and Q' as a function of moisture content, the numerical model can handle transient-state moisture conditions provided the moisture content does not change too rapidly. Such data at this point, however, are not available.

The major need at present for determining the effects of the rhizosphere on nutrient uptake for specific plants is an accurate knowledge of the rhizosphere transport properties. Given accurate knowledge of these properties as a function of time, space, and concentration, the numerical model should be useful in simulating nutrient depletion in the soil system. This information may in turn lead to more efficient fertilization practices and the breeding of plants with better nutrient uptake characteristics.

The model can be used to estimate the required accuracy of input data to show effects on nutrient depletion.

APPENDIX

It was brought to the author's attention by a reviewer that there can be several definitions for a buffer power. I will discuss the relationship between two of these definitions. For steady-state (or nearly steady-state) moisture conditions in a nondeforming medium, we may assume

$$C_T = f(C_i) \quad (A1)$$

where C_T is the total concentration and C_i is the liquid phase concentration. In this case we may define a buffer power by

$$b(r, t) = \frac{\partial C_T}{\partial C_i} \quad (A2)$$

This gives rise to

$$\frac{\partial C_T}{\partial t} = \frac{\partial C_T}{\partial C_i} \frac{\partial C_i}{\partial t} = b(r, t) \frac{\partial C_i}{\partial t} \quad (A3)$$

In this manuscript, b is used in the above sense. We can also define a buffer power, b' , by

$$C_T = \left[\frac{\rho S(C_I)}{C_I} + \theta \right] C_I = b' C_I \quad (\text{A4})$$

where ρ is the bulk density, S is the ratio of the mass of ion absorbed to the mass of dry soil, and θ is the volumetric moisture content. The reader should note that Eq. (A4) does not require a constant bulk density or nearly steady-state moisture conditions. From Eq. (A4) we have

$$\frac{\partial C_T}{\partial t} = \frac{\partial b'}{\partial t} C_I + b' \frac{\partial C_I}{\partial t} \quad (\text{A5})$$

Thus for nearly steady-state moisture conditions in a nondeforming medium, we find

$$\frac{\partial b'}{\partial t} = \frac{\partial b'}{\partial C_I} \frac{\partial C_I}{\partial t} \quad (\text{A6})$$

which gives

$$b = \frac{\partial b'}{\partial C_I} C_I + b' \quad (\text{A7})$$

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